

Title

Nonlinear precoding method for a digital broadcast channel

Description

The invention relates to a nonlinear precoding method based on a modulo arithmetic for the transmit-side preequalization of K user signals to be transmitted at the same time and frequency in a digital broadcast channel with known transmission behavior set up between a central transmitting station and K decentralized, non-interconnected receiving stations, said user signals consisting of data symbols a_k with k from 1 to K from an M_k -level signal constellation having a signal point spacing A_k with a periodic multiple representation of the undisturbedly transmitted data symbols a_k in data symbol intervals congruent for K receive-side modulo decision devices, a transmit-power-minimizing selection of representatives v_k from the range of values $a_k + A_k \cdot M_k \cdot z_{kk}$ where z_{kk} is from the set of integers, and linear preequalization of the selected representatives v_k to form transmit signals x_k to be transmitted.

In a broadcast channel, a plurality of user signals present at a common (i.e. central) transmitter (e.g. a base station) are digitally transmitted to a plurality of decentralized (i.e. scattered over a service area) receivers (e.g. mobile stations). Signal transmission user signal \rightarrow receive signal is unidirectional in the downlink. The particular feature of signal transmission in a broadcast channel is the lack of cooperability between the individual receivers. At no receiver are the signals of the other receivers known, and communication between the individual receivers is not possible. Consequently there can be no joint data processing

of the receive signals in a central receiver. Transmission-improving signal conditioning can therefore only take place at the transmit side in the common transmitter. Signal transmission can be wireline, but tends to be non-wireline. The essential but imperfect differentiation of the signals for correct assignment of each user signal to the associated receiver is performed by Code Division Multiple Access (CDMA) or by Space Division Multiple Access (SDMA). The resulting overall structure with a large number of signal inputs (user signals) and a large number of signal outputs (receive signals) is known as a MIMO system (Multiple Input Multiple Output). Moreover, in the case of non-wireline signal transmission (radio transmission), multi-antenna systems are being increasingly used in which the signals are transmitted via a large number of transmitting antennas to a large number of receiving antennas, the numbers of antennas possibly being the same or different and having an impact on signal processing. In general, time and space diversity can be advantageously utilized in a MIMO system.

The problem arising from a plurality of receivers being supplied from a common transmitter is that the individual users are supplied not only with their own wanted signals, but that other users' signals are superimposed thereon, resulting in interference signals. The occurrence of crosstalk interferences is synonymous with loss of the orthogonality which would be present in the case of ideal transmission behavior with decoupled subchannels. On the transmit side it must therefore be attempted, knowing the user signals and the transmission conditions currently obtaining in the broadcast channel, i.e. the individual crosstalk factors between the individual users, to generate a suitable common transmit signal in such a way that each user receives his desired signal but without interference from the other signals. In

contrast to the twin problem of the multiple access of scattered transmitters to a common receiver (uplink) for which many approaches are now known, the literature only contains a small number of methods for solving the described problem of serving spatially separated, non-cooperating receivers from a common transmitter. The described transmission scenario can be expressed in a mathematically compact and general manner using the well-known channel equation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

The possibly already preprocessed transmit symbols of the K users are combined in the vector $\mathbf{x} = [x_1, x_2, \dots, x_K]^T$ (vector and matrix notation in bold). The complex-valued elements h_{ki} of the channel matrix \mathbf{H} describe the couplings between the transmission paths $i \rightarrow k$, i.e. the crosstalk of the user i onto the user k . The ideal channel matrix \mathbf{H} without couplings is a diagonal matrix, preferably an identity matrix (value 1 on the main diagonal). The channel matrix \mathbf{H} can be estimated by various known methods with backchannel or, in the case of duplexing with time division multiplex, also without backchannel and is assumed to be known at the central transmitter (presence of the so-called Channel State Information CSI). Combined in the vector \mathbf{n} are the unavoidable noise effect (additive noise) of the electronic components involved and other external interference, and the elements of the vector $\mathbf{y} = [y_1, y_2, \dots, y_K]^T$ are the receive symbols at the individual receivers. The first known approach for a broadcast channel involves linear preequalization of the user signals (Linear Channel Inversion LCI). From the users' data signals a_k present, combined in the vector \mathbf{a} , the transmit symbols x_k (the term "symbol" in this context means a real or complex number representing the information) are formed according to

$$\mathbf{x} = \mathbf{H}^{-1}\mathbf{a}$$

where \mathbf{H}^{-1} represents the inverse matrix to \mathbf{H} , which can only be formed, however, if the transmission matrix is non-singular (determinant of the matrix is non-zero). By these means it is achieved, however, that no interference signals are produced at the receivers and the data symbols a_k appear directly (with only additive noise superimposed). There is therefore complete decoupling of the individual direct transmission paths $k \rightarrow k$ (orthogonality). However, the disadvantage of this procedure is the associated, in some cases very substantial, increase in the average transmit power required. This effect is greater the more strongly the matrix \mathbf{H}^{-1} tends to a singular matrix.

A significant increase in the average transmit power is avoided if, instead of linear preprocessing of the user signals, joint nonlinear preequalization (precoding method) is used. With the known precoding methods, however, the mutual interference signals are likewise completely suppressed, so that diversity reception cannot be utilized. Precoding methods can be developed from the twin problem to this situation, i.e. the multiple access scenario (multiple access problem e.g. in the uplink transmit direction in which a plurality of distributed users access a common receiver). There, nonlinear equalization can be performed by successive elimination of the interference signals which is implemented, for example, in the known V-BLAST method and can be termed Zero Forcing Decision Feedback Equalization (ZF-DFE) completely eliminating (Zero Forcing ZF) the interference signals. An established precoding method is known according to Tomlinson and Harashima (THP - Tomlinson-Harashima Precoding) and is based on the use of modulo arithmetic. This procedure is described for the first time by M. Tomlinson in **publication I** "New Automatic Equaliser Employing Modulo Arithmetic" (Electronics Letters, vol. 7,

Nos. 5/6, pp. 138-139, March 1971) and by H. Harashima and H. Miyakawa in **publication II** "Matched Transmission Technique for Channels with Intersymbol Interference" (IEEE Transactions on Communications, Vol. com. 20, No. 4, pp. 774-780, August 1972). Originally the nonlinear precoding methods were only designed for channels with one input and one output, but with intersymbol interference (ISI) present. It was later recognized that they could also be used on MIMO channels in order to suppress interchannel interference (ICI) or a combination of ISI and ICI. This transmission is described in detail, with the coining of the term MIMO precoding, in **publication III** by R. Fischer et al. "Spacetime Transmission using Tomlinson-Harashima-precoding" (Proceedings of 4. ITG Conference on Source and Channel Coding, pp. 139-147, Berlin, January 2002).

This preequalization can be used in the central transmitter instead of receive-side feedback equalization which is only possible in the case of a central receiver. To ensure that the average transmit power is not significantly increased in the process, THP operates on a nonlinear basis. Here modulo reduction with a sawtooth characteristic limits the transmit signal x_k to the range $(-M_k/2, +M_k/2]$ at a level number M_k of the signal constellation selected for the relevant data symbol a_k and a selected signal point spacing $A_k = 1$. It should be pointed out at this juncture that in principle a separate level number M_k and a separate signal point spacing A_k can be selected for each data stream to be transmitted. In general, however, for the sake of simplicity these parameters are selected identically for all the user signals to be transmitted. For any given data signals, the output signal is constantly held between predefined modulo limits by means of a simple addition rule, by which means the transmit power can be significantly reduced compared to linear methods. This

limiting is performed symbol-by-symbol without memory and is equivalently representable as the addition of a correction symbol which may assume an integral multiple of $A_k \cdot M_k$. The now apparently linear preequalization in this approach completely nullifies the channel distortion. Essentially with THP, by multiple representation of the data symbols a_k and selection of suitable representatives v_k which are then linearly preequalized, the transmit signal is therefore formed according to $\mathbf{x} = \mathbf{H}^{-1}\mathbf{v}$ so that any appreciable increase in the average transmit power can be avoided. By means of the multiple representation and selection of a suitable representative v_k , one more degree of freedom is therefore provided for signal processing. In the case of binary transmission, the binary symbols "0" and "1" can be represented e.g. by the amplitude values of +0.5 and -0.5 (signal point spacing $A_k = 1$), corresponding to an $M_k=2$ -level signal constellation. On the basis of the amplitude values selected, when using precoding the binary symbol "0" can, for example, be represented by .. -3.5; -1.5; +0.5; +2.5; +4.5; . . and the binary symbol "1" by . . -2.5; -0.5; +1.5; +3.5; +5.5; ... with a respective addition of an integer (multiple of $M_k = 2$). With knowledge of all the user data symbols a_k (having the values +0.5 and -0.5), the representatives v_k (from the range of values $(+0.5 + 2z)$ where z is a positive or negative integer) are then selected such that, after linear preequalization of the channel, the transmit signal $\mathbf{x} = \mathbf{H}^{-1}\mathbf{v}$ possesses a low average power or smallest possible amplitude.

The prior art on which the present invention proceeds is disclosed in **publication IV** of R. Fischer et al.: "MIMO-Precoding for Decentralized Receivers" (Proceedings of International Symposium on Information Theory - ISIT 02, Lausanne, Switzerland, June/July 2002, p. 496). In continuation of publication III, a modified THP using

nonlinear modulo arithmetic is described for a broadcast channel with downlink scenario in which the decentralized receivers have no contact with one other. The transmit-side nonlinear preprocessing can be derived from DFE and has, in mathematical terms, a unitary matrix \mathbf{F} operated in the forward direction whose function is to transform the channel matrix into triangular form, and a matrix \mathbf{B} present in the nonlinearly operating feedback loop in the form of a lower triangular matrix with unit main diagonal. If the overall channel matrix for the transmission behavior is of triangular form, the interference signals occurring can be precompensated bit by bit in the feedback branch of the central transmitter using modulo arithmetic. At the individual receivers, the data then appears as if the other users (with parallel transmission paths to the other receivers) did not exist.

As mutual interference signals are therefore completely avoided also when using nonlinear precoding methods, in each receiver the transmitted data symbols a_k can be recovered or estimated values for them can be formed by means of threshold decision-making which takes account of the periodic continuation of the amplitude values or signal point spacings (modulo decision device). However, the disadvantage of these precoding methods is that no "diversity gain" can be achieved because of the complete prevention of mutual interference signals. Each transmission subsystem (one user signal to the associated receiver) functions as if it is operated via a separate channel (with one input and output). Specifically in the case of fading channels this involves a high error rate at times of poor transmission conditions. However, if signals are jointly processed and transmitted, a diversity gain can in principle be achieved. If in the case of two transmission paths one of them has poor transmission conditions, it is

highly probable that the other transmission path is quite usable.

On the basis of **publication VI** it is therefore the object of the present invention to further develop a nonlinear precoding method of the generic type for a broadcast channel with decentralized receivers in such a way that its available diversity due to interference signals present can be utilized. The method is designed to be simple in its execution and to ensure high transmission quality. The inventive solution for achieving this object is set forth in the main claim. Advantageous developments of the nonlinear precoding method according to the invention are disclosed in the sub-claims. These will now be explained in greater detail in connection with the invention.

The precoding method according to the invention for transmit-side, joint preprocessing of the user signals is based on the above described THP and returns to the idea of periodic continuation of the possible representatives for the individual data symbols a_k , the mutually superimposed interference signals no longer being zero forced but being included by means of the joint preprocessing by assigning values from a precisely defined set of values to these interference values also, the values permitted for characterizing the interference signals being selected such that the receive-side modulo decision devices can still decide reliably on the transmitted data symbol a_k even with the interference signals present. Since when using THP the receive-side modulo decision devices already allow for a periodic ambiguity of the data symbols a_k , the interference signals may now assume values which mean that a different representative v_k from the possible set of representatives, but which represents the same data symbol a_k , appears at the

receiver, the range of values $(a_k + A_k \cdot M_k \cdot z_{kk})$, where z_{kk} is a positive or negative integer including zero) for the data symbols transmitted undisturbedly from user k to receiver k differing by just the original data symbol a_k from the range of values $(A_k \cdot M_k \cdot M_k \cdot z_{Ik})$, where z_{Ik} is a positive or negative integer including zero) for the superimposed interference signals from user I to receiver k , the subscriber k being excluded. With the precoding method according to the invention, interference signals present, by producing a permissible shifting of the modulo-coded user signals into decision intervals for identical data symbols, are therefore taken into account and co-processed. Although the periodic shifting means that the decision intervals are different, the result of the decision and its reliability are identical.

In the above described numerical example for binary data transmission, the mutual interference signals may therefore assume the values...-4; -2; 0; +2; +4;... (even numbers) and therefore be even-numbered. However, the mapping of the interference signals to multiples of whole numbers also applies to any other selection of M_k -level signal constellations. The precoding according to the invention can also be applied to the quadrature amplitude modulation schemes (e.g. 4QAM or 16QAM) using complex number space. In this kind of modulation the data vector **a** consists of data symbols a_k (also known as signal points) from a complex-valued QAM alphabet. The real parts of the symbols are transmitted with a cosine wave modulation and the imaginary parts with a sine wave modulation (quadrature mixing). When using complex-valued channel descriptions (complex-valued matrix entries) it is advisable first to split the entries up into two real-valued components, i.e. real and imaginary part and represent them as real transmission using twice as many sub-signals. The channel matrix **H_r** therefore attains twice the dimension (2K). On the

receive side, demodulators are provided which detect the voltage values phase-correctly and re-assign the real components real and imaginary part. Then finally transformation back to the complex-valued space again takes place. We get:

$$\begin{bmatrix} \text{Re}\{y\} \\ \text{Im}\{y\} \end{bmatrix} = \begin{bmatrix} \text{Re}\{H\} & -\text{Im}\{H\} \\ \text{Im}\{H\} & \text{Re}\{H\} \end{bmatrix} \cdot \begin{bmatrix} \text{Re}\{x\} \\ \text{Im}\{x\} \end{bmatrix} + \begin{bmatrix} \text{Re}\{n\} \\ \text{Im}\{n\} \end{bmatrix}$$

where Re and Im denote the real and the imaginary part of the relevant variable and define, according to the definition selected, an equivalent 2K-dimensional, real-valued (MIMO) channel model according to:

$$\mathbf{y}_r = \mathbf{H}_r \mathbf{x}_r + \mathbf{n}_r.$$

In order to simplify the equalization and advantageously utilize the possible diversity, with the precoding method according to the invention the broadcast channel present is notionally subdivided into two sections. The first part is completely equalized by means of precoding, the user signals are therefore decoupled, but a periodic continuation of the data symbols takes place. The current representative v_k for the data symbols a_k is selected on an ongoing basis from the possible values which differ by integral multiples of the original level number M_k , so that after linear preequalization of the selected representatives v_k the required transmit power is minimal. The second part of the channel is not equalized and therefore produces residual interference between the user signals. By suitably selecting this part, however, it can be achieved that, on the one hand, the residual interference is so constituted that it does not adversely affect decision making in the receivers and, on the other, that equalization

of the first part of the channel is possible with lower receive-side gain and therefore lower noise amplification or the diversity of the channel can be at least partially utilized. Since when using preequalization methods the user signals already appear periodically continued at the receivers, the residual interference may assume values coinciding with the spacing of the possible representatives, the interference being reflected only in the (virtual) selection of a different representative, and its effect being completely eliminated in the modulo decision device already present. The crucial advantage of the invention is the greatly increased power efficiency of signal transmission. By means of the precoding method claimed, a lower bit error rate, i.e. reliable reception, can be achieved at the same average transmit power as with the known precoding methods. In particular, using the precoding method claimed, a diversity gain can be obtained which makes itself positively felt in a more rapid reduction in the bit error rate as the transmission quality of the broadcast channel improves.

A partial equalization of the transmission channel corresponds to using a changed channel description, in mathematical terms converting the channel matrix \mathbf{H}_r to a reduced channel matrix \mathbf{H}_{red} . Its multiplication with a suitable residual interference matrix \mathbf{R} describing the remaining mutual couplings then yields once again the channel matrix \mathbf{H}_r according to the decomposition

$$\mathbf{H}_r = \mathbf{R} \mathbf{H}_{red}.$$

The residual interference matrix \mathbf{R} is only dependent on the current transmission behavior of the channel. As long as the channel matrix \mathbf{H} (or \mathbf{H}_r) does not change (burst transmission), the residual interference matrix \mathbf{R} does not change either. On the main diagonal the residual interference matrix \mathbf{R} is

occupied by ones (direct signal paths), all the other elements assume row-wise only integral (positive or negative) multiples of the level number M_k . In the case of binary transmission per component, these elements are only even (positive or negative) numbers:

$$\mathbf{R} = \begin{bmatrix} 1 & & 2\mathbf{Z} \\ & \ddots & \\ 2\mathbf{Z} & & 1 \end{bmatrix}$$

Various methods can be used for suitable decomposition of the channel matrix \mathbf{H} . By using the Monte Carlo method, the appropriate coefficients can be determined e.g. empirically. Optimum decomposition methods must be selected for their complexity, which should be as low as possible, and for required receive-side gain that is as small as possible (e.g. by means of Automatic Gain Control AGC).

Other matrix reductions for MIMO channels are known from the prior art as lattice reduction. In **publication V** by H. Yao et al.: "Lattice-Reduction-Aided Detectors For MIMO Communication Systems" (Proceedings of IEEE Globecom 2002, Taipei, Taiwan, November 2002) there is described a low-complexity detection method for channels with a plurality of inputs and outputs. The basic idea is the use of mathematical methods which are known from the field of lattice theory (theory of regular lattices), the MIMO channel not being linearly equalized completely but, on the basis of another, more suitable representation (reduced basis), the channel is only partly equalized so that a simple component-wise (i.e. in respect of the individual user signals) threshold decision is possible. Only after further postprocessing are the required estimated values for the transmitted data signals obtained. However, the

known method specifically considering the case of two transmitting and two receiving antennas differs fundamentally from the invention in that there, in a multi-antenna system, all the user receive signals are known on the receive side and joint signal processing is possible. In contrast to this, the precoding method according to the invention relates to the problem in a multiuser system with exclusively downlink direction. Here a common transmitter is present at which all the user signals are known and can be processed. On the other hand, the receivers scattered over a service area cannot cooperate, i.e. each receiver sees only its own receive signal (no joint processing possible). The known (partial) equalization takes place exclusively on the receive side on an exclusively linear basis, i.e. the reduced portion of the channel is equalized using the inverse channel matrix. The invention operates exclusively nonlinearly on the transmit side on the basis of THP.

Publication VI of Ch. Windpassinger and R. Fischer: "Low-Complexity Near-Maximum-Likelihood Detection and Precoding for MIMO Systems using Lattice Reduction" (Proceedings of IEEE Information Theory Workshop 2003, pp. 345-348, Paris, France, March/April 2003) is based on and expands **publication V**. The low-complexity detection method for MIMO channels with a plurality of inputs and outputs is extended from the 2x2 scenario to the general case of K inputs and outputs. In addition, the linear partial equalization is replaced by a nonlinear precoding. However, the critical difference from the invention is that these methods again relate to multi-antenna systems in which all the partial receive signals are known on the receive side and joint processing is possible. The preequalization method claimed with the present invention is, on the other hand, designed for decentralized receivers that cannot cooperate, wherein lies a particular difficulty for

signal processing, because it can only take place on the transmit side.

To equalize the channel portion, described by the reduced portion \mathbf{H}_{red} , this reduced matrix is further factorized into suitable matrices. This decomposition can have e.g. the following form:

$$\mathbf{P}^T \mathbf{H}_{\text{red}} = 1/g \mathbf{B} \mathbf{F}^{-1},$$

where \mathbf{F} is a matrix with orthogonal columns, \mathbf{B} is the lower triangular matrix, \mathbf{P} a permutation matrix (each row and each column contains a single 1), and g the receive-side gain factor (automatic gain control). All three matrices and the scalar can be unambiguously determined from \mathbf{H}_{red} according to a predefined criterion (preferably minimum g).

For normal selection of the signal points from the array of integers shifted by $\frac{1}{2}$ in the numerical example given above, a systematic offset is produced on the receive side. This can be eliminated either by correspondingly modified receivers or more simply by transmit-side offset compensation for which no additional transmitting energy is required. This takes place by subtraction of the vector \mathbf{o} according to

$$\mathbf{o} = \mathbf{P}^T (\mathbf{R} - \mathbf{I}) [1/2 \dots 1/2]^T.$$

Embodiments of the invention will now be explained in greater detail with reference to the schematic diagrams in which

Figure 1 shows a broadcast channel,

Figure 2 shows the decoupling of the broadcast channel by means of a prior art precoding method,

- Figure 3** shows the broadcast channel with the precoding method according to the invention superimposed on it,
- Figure 4** shows a block diagram of the precoding method according to the invention,
- Figure 5** shows bit error curves for various equalization methods and
- Figure 6** shows the gain factors for various equalization methods.

Figure 1 schematically illustrates the structure of a broadcast channel **BC** for digital communication of K user signals \mathbf{ST}_k from a common, central transmitter **CT** (e.g. a base station) to K decentralized receivers \mathbf{DR}_k (e.g. mobile stations) which shall in each case only receive their own receive signal \mathbf{SR}_k and have no contact with the adjacent receivers \mathbf{DR}_k . Transmission takes place exclusively in the downlink direction, non-wireline radio transmission being used in the case illustrated. The broadcast channel **BC** considered in its entirety has a large number of inputs and a large number of outputs and can therefore be interpreted as a MIMO channel (Multiple Input Multiple Output). A multiuser system is present here which must be differentiated from a multi-antenna system which likewise defines a MIMO channel.

Figure 2 shows for a selected exemplary embodiment with $M_k=2$, $A_k=1$ and $k=1, \dots, K$ the completely decoupled broadcast channel **BC** on the basis of applying the known nonlinear precoding method **THP** (Tomlinson-Harashima-Precoding) using modulo arithmetic which has already been explained above. Basically there is added to each data symbol a_k (assigned to the user signals \mathbf{ST}_k) a special value of an integral multiple of the product of the level number M_k and the signal point spacing A_k

of the signal constellation ($A_k \cdot M_k \cdot z$ where z is a positive or negative integer including zero) and the best value in respect of minimum transmit power is selected and the signal representative thus obtained is linearly preequalized. **THP** is used on the transmit side to produce in the central transmitter **CT** a common transmit signal such that each decentralized receiver **DR_k** receives its required receive signal **SR_k**. Interference signals present are completely eliminated with this precoding method **THP** so that channel diversity cannot be used.

Figure 3, on the other hand, illustrates the application of the nonlinear precoding method according to the invention, taking interference signals into account. With this method the interference signals in the case of binary (in the exemplary embodiment shown with $M_k=2$, $A_k=1$ and $k=1, \dots, K$) transmission with even, integral values between the values for the user signals **ST_k** are multiply represented, the interference symbols between the data symbol a_I (with I from 1 to K and not equal to k) and the data symbol a_k being assigned periodic representatives from the value range $A_k \cdot M_k \cdot z_{Ik}$ where z_{Ik} is from the set of integers. The mapping of the interference signals to even, integral values (even-numbered interferences) can be appropriately shortened using **EIIP** (Even-Integer Interference Precoding). In **Figure 3** the basic principle of partial channel equalization on which **EIIP** is based can be clearly seen, whereby the broadcast channel **BC** is virtually [converted] into a reduced channel without coupling (first addition positions) which undergoes nonlinear precoding (shown in linearized form), and a superimposition of the suitably formed interference signals (second addition positions) is discriminated.

Figure 4 (top) shows the entire transmission system as it is provided in the proposed partially equalizing precoding method **EIIP**. The channel matrix \mathbf{H} denotes the actual transmission channel with K users. At its input, all the transmit signals can be jointly accessed, which is indicated by a wide vector arrow. At its output, the user signals y_k with k from $1 \dots K$ are only processed singly, here indicated by individual scalar arrows. On the receive side, further noise n_k is superimposed. In the normal abstract representation shown, the receivers each consist only of a scaling device (Automatic Gain Control) and a threshold decision device (indicated in **Figure 4** by a g in the circle and a double-bordered box with threshold decision, the double-border standing for a nonlinear operation). The transmitter consists of the first three functional blocks. This involves a permutation matrix \mathbf{P}^T depending on the existing channel matrix \mathbf{H} (or \mathbf{H}_{red}), a feedback loop with a nonlinear modulo operation **MOD**, the identity matrix \mathbf{I} and a lower triangular matrix \mathbf{B} as well as a matrix \mathbf{F} with orthogonal columns. The data symbols to be transmitted (taken from a QAM alphabet) are combined in the K -dimensional vector \mathbf{a} . Each receiver wishes to receive its data symbol a_k (and that alone). This vector with complex entries is first converted into a real vector (separation of the complex components into real and imaginary part as already described above), symbolized by the notation \mathbf{a}/\mathbf{a}_r . The further processing in the transmitter takes place on a real-value basis. The transmitter produces transmit symbols, combined in the vector \mathbf{x}_r . These are then translated to a complex-valued representation (combination of real and imaginary part to form a complex number; reverse process as above), as the channel processes complex-valued input symbols.

The first stage of the transmitter is a permutation (re-sorting) of the components of the vector \mathbf{a}_r . The next

functional block is the nonlinearly operating feedback loop known in precoding methods. Here the interference signals occurring during transmission over the channel are already pre-compensated. In order not to increase the transmit power, a modulo operation **MOD** is used here which limits the output symbols to a fixed predefined interval by addition/subtraction of a suitable integral value (corresponding to a periodic continuation of the original signal constellation). All the signal points possessing a predefined spacing $A_k \cdot M_k$ from one another, e.g. in the case of binary transmission ($M_k=2$) and signal point spacing $A_k=1$ equivalent to $A_k \cdot M_k=2$, represent the same message (bit combination). Finally another unitary matrix **F** is applied which converts the general channel matrix into a lower triangular matrix without increasing the transmit power. Only thus can successive processing, as required, take place in the transmitter.

To ensure that the receive signals appear without offset ϕ at the receivers, this is already pre-compensated in the transmitter. The matrices **P**, **B** and **F** are computed uniquely from the reduced form of the channel matrix as described above. The precoding therefore equalizes only this reduced portion; the interference signals due to the residual interference matrix **R** (see above) remain.

The mode of operation of transmission is illustrated in the middle and bottom row in **Figure 4**. First the precoding loop is replaced by its linearized representation. The modulo operation is replaced by the addition of a correction term **d**. The remaining, linear feedback loop (forward transmission **One**; feedback **B-I**) is then realized precisely by the matrix **B**⁻¹ (inverse matrix of **B**). The channel matrix is represented, as described in the exemplary embodiments above, as a cascade of the reduced channel matrix **H_{red}** and the residual interference

matrix \mathbf{R} . Because of the specific construction of the matrices \mathbf{B} and \mathbf{F} from \mathbf{H}_{red} , the cascade of \mathbf{B}^1 , \mathbf{F} and \mathbf{H}_{red} produces precisely the matrix \mathbf{P}/g (again above equation), thereby producing the structure shown in the bottom row. The permutation matrices \mathbf{P}^T and \mathbf{P} cancel each other out; as transmission matrix, there therefore remains only the residual interference matrix \mathbf{R} . This describes the interferences (couplings) between the user signals. As the main diagonal is one, the wanted signals are transmitted ideally. The secondary diagonal elements which describe the crosstalk between the users, are even-numbered in the case of binary transmission; only even-numbered interferences therefore occur. However, this does not impair the existing modulo decision.

Figure 5 shows the average bit error curves of the users for various signal processing methods. The average bit error rate BER in each case is plotted against the ratio (expressed in dB) of the average transmit energy E_b per information bit to the spectral power density N_0 of the additive noise. Two users ($K=2$) are assumed, which occurs relatively often if, for example, in addition to a large number of users with low data rates and transmit powers there are two users with high data rates and transmit powers to which preprocessing is then limited. On the basis of the method selected, specifically the decomposition of the reduced channel matrix \mathbf{H}_{red} into g , \mathbf{F} , \mathbf{B} and \mathbf{P} , the same bit error characteristic is produced for both users. The most favorable bit error characteristic is provided by joint signal processing at the receiver (joint processing at receiver **JPR**, **curve a**), the least favorable by purely linear channel inversion on the transmit side (linear preequalization **LPE**, **curve b**). Increasingly favorable error behavior is then shown by the transmission channel with transmit-side nonlinear complex-valued precoding (**CVP**, **curve c**) and real-valued precoding (**RVP**, **curve d**). The error

behavior coming nearest to receive-side joint signal processing is achieved using the precoding method according to the invention with partial channel equalization taking interference into account (**EIIP, curve e**).

Due to the significantly improved error behavior with the nonlinear precoding method according to the invention **EIIP**, much lower gain factors are required on the receive side despite the minimized transmit power. For a transmission system with two users ($K=2$), **Figure 6** plots the gain factors $g_{\text{EIIP-PREC}}$ using nonlinear precoding according to the invention against the gain factors g_{PREC} which arise with a real-valued precoding method not taking account of the interference (the representation is in dB as the inverse of the square, as the signal-to-noise ratio SNR is proportional to this term and this term directly describes the capability of the method). The magnitude of the gain factor g_{PREC} is plotted on the x-axis and is also identifiable via the straight line $y = x$ as the lower end of the bar. The corresponding gain factors $g_{\text{EIIP-PREC}}$ are shown as the upper end of the bar. The length of the bar then indicates the achievable gain. Large gains are apparent particularly in situations in which the known methods produce very poor results.

Reference character list

A_k	Signal point spacing for the transmit symbol a_k
AGC	Automatic Gain Control
a	Data vector
a_k	kth data symbol with k from 1...K
a_r	Real-valued data vector
B	Lower triangular matrix with 1 on the main diagonal
BC	Broadcast channel for digital communication
BER	Bit error rate
CT	Central transmitter
DR_k	Decentralized receiver
E_b	Average transmit energy per bit
EIIP	Even-Integer Interference Precoding
F	Unitary (orthogonal) matrix
g	Gain factor
H	Channel matrix
H_{red}	Reduced channel matrix
I	Identity matrix
I, k	User subscript
K	Number of users or receivers
M_k	Signal constellation level number for transmit symbol a_k
MIMO	Multiple Input Multiple Output
MOD	Nonlinear modulo reduction
n_k	kth noise signal with k from 1...K
o	Offset
P	Permutation matrix
PREC	Precoding method
QAM	Quadrature Amplitude Modulation
R	Residual interference matrix
SR_k	Receive signal
ST_k	User signal
THP	Tomlinson-Harashima Precoding
x_r	Real-valued transmit vector

y_k k th user signal with k from 1... K

z Positive or negative integer including zero